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Analysis of the Panoptic protocol

Formal analysis of the Panoptic protocol

Samuel Cohen[1], Marc Sabaté Vidales[1], David Siska [1], and Łukasz Szpruch [1]

[1]Simtopia.ai

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2 Overview

The Panoptic protocol provides a mechanism for entering a variety of long and short margined positions on Uniswap v3; through the mechanism of entering into Panoptions. Conversely, Panoptions can be thought of as perpetual options, with a payoff function based on the value of the assets that a liquidity provision position in Uniswap v3 holds.

It has been shown empirically [5] and experimentally [9] that liquidity providers (LPs) in constant function markets (in particular, Uniswap v3) lose money on average. Indeed, from e.g. [5, Table 2], the average LP transaction in the ETH/USDC pool (from May 2021—August 2022) resulted in a position loss of -1.64%, and fee income of 0.155% of the size of the initial trade, with an average hold time of 6.1 days. This agrees broadly with the analysis in [6], which indicates that holding a Uniswap LP position is less profitable than a corresponding options position, with the associated streaming premium. We provide more details in Section 6.

As we will show in Section 4.1, there is a theoretical, arbitrage-free level of fee income that would make Uniswap LP positions fair. The Panoptic protocol, by allowing leveraged and short positions in Uniswap v3, can provide a venue for inefficiencies to be exploited and closed, provided those who enter short-put Panoptions (moving liquidity from Panoptic to Uniswap v3) are compensated with appropriate arbitrage-free streaming premia, which will be paid by those who take more general Panoption positions (moving liquidity from Uniswap v3 back to Panoptic).

The first challenge for the Panoptic protocol is to determine streaming premia which will close this statistical arbitrage. The difficulty is that this needs to be done without exposing Panoptic LPs to risk (which ensures that Panoptic LPs don't suffer from this statistical arbitrage), and to encourage traders to preferentially trade through the Panoptic protocol, through which they additionally benefit from leveraged LP provision.

Given the Panoptic protocol also allows leveraged positions, it is necessary that these be risk managed appropriately. This leads to the importance of appropriate margin calculations and close-out protocols for market participants, to ensure counterparty risk is well managed in a decentralized way, and the risks to different participants in the protocol are well understood.

3 Liquidity provision in constant function markets

In this section, we will draw a connection between a constant function market (CFM) and a perpetual (American) options contract. We will particularly seek to derive the impermanent loss (also known as LVR) of a CFM, or equivalently the arbitrage-free streaming premium of the perpetual option; this is the natural analogue of the θ sensitivity of European options[1].

[1] [7] uses an approximation of the Uniswap-implied perpetual American payoff in terms of the Black–Scholes value of a covered European call option in order to obtain a variant of the

3.1 CFM – an overview

A constant function market (CFM) is characterised by

i) The reserves $(x^1, x^2) \in \mathbb{R}^2_+$ describing amounts of assets in the pool.

ii) A constant function market $\Psi : \mathbb{R}^2_+ \to \mathbb{R}$ which determines the state of the pool after each trade according to the acceptable fund positions:

$$\left\{(x^1, x^2) \in \mathbb{R}^2_+ \,:\, \Psi(x^1, x^2) = \text{constant}\right\}.$$

iii) A trading fee $(1-\gamma)$, for $\gamma \in (0, 1]$.

For the purposes of this report, we assume $\Psi : \mathbb{R}^2_+ \to \mathbb{R}$ to be twice continuously differentiable and convex (see [4]).

To acquire Δx^1_t of asset x^1 at time t a trader needs to deposit a quantity $\Delta x^2 = \Delta x^2(\Delta x^1)$ of asset x^2 into the pool, and pay a fee $(1-\gamma)\Delta x^2$.[2] Δx^2 and Δx needs to satisfy the equation

$$\Psi(x^1_t - \Delta x^1, x^2_t + \Delta x^2) = \Psi(x^1_t, x^2_t).$$

Once the trade is accepted the reserves are updated according to

$$x^1_{t+1} = x^1_t - \Delta x^1 \quad \text{and} \quad x^2_{t+1} = x^2_t + \Delta x^2.$$

The relative price of trading Δx^1 for Δx^2 is defined as

$$\frac{P^{1,CFM}_t(\Delta x^1)}{P^{2,CFM}_t(\Delta x^2)} := \frac{\Delta x^2}{\Delta x^1} \quad \text{subject to} \quad \Psi(x^1_t - \Delta x^1, x^2_t + \Delta x^2) = \Psi(x^1_t, x^2_t).$$

Observe that

$$\begin{aligned} 0 =& \Psi(x^1_t - \Delta x^1, x^2_t + \Delta x^2) - \Psi(x^1_t, x^2_t) \\ =& -\partial_{x^1}\Psi(x^1_t, x^2_t)\Delta x^1 + \partial_{x^2}\Psi(x^1_t, x^2_t)\Delta x^2 + \mathcal{O}((\Delta x)^2). \end{aligned}$$

Hence the relative price of trading an infinitesimal amount of Δx^1 for Δx^2 is given by

$$\frac{P^{1,CFM}_t}{P^{2,CFM}_t} := \lim_{\Delta x^1 \to 0} \frac{P^{1,CFM}_t(\Delta x^1)}{P^{2,CFM}_t(\Delta x^2)} = \frac{\partial_{x^1}\Psi(x^1_t, x^2_t)}{\partial_{x^2}\Psi(x^1_t, x^2_t)}.$$

Assume that there is an external market where assets x^1 and x^2 can be traded (without frictions) at the prices $S_t = (S^1_t, S^2_t)$. The no-arbitrage condition in the case of no fees ($\gamma = 1$) implies that

$$\frac{P^{1,CFM}_t}{P^{2,CFM}_t} = \frac{S^1_t}{S^2_t} \tag{1}$$

streaming premium as the Black–Scholes θ. This is arguably a formally unnecessary step, as the streaming premium is well defined for the perpetual directly, without needing the Black–Scholes model assumptions; even though the connection to classical Black–Scholes theory may be useful for understanding of the protocol's construction and implications.

[2] The fee in Uniswap-V3 is not added to the pool reserves [1]. This is in contrast to Uniswap-V2.

Conversely, if 1 would not hold, then (in amarket with no fees) there would be an arbitrage opportunity between CFM and the external market (assuming frictionless trading is possible), as it would be possible to purchase a combination of assets cheaply in one market, and then sell it in the other.

Example 3.1 (GMM)**.** *Consider the trading function*

$$\Psi(x^1, x^2) = (x^1)^\theta (x^2)^{1-\theta}$$

for $\theta \in (0,1)$*. In the setting with no fees,* $\gamma = 1$*, we have* $(x_t^1)^\theta (x_t^2)^{1-\theta} = (x_0^1)^\theta (x_0^2)^{1-\theta}$*. The no arbitrage relationship (1), in GMM is given by*

$$\frac{P_t^{1,CFM}}{P_t^{2,CFM}} = \frac{\theta x_t^2}{(1-\theta)x_t^1} = \frac{S_t^1}{S_t^2}\,. \tag{2}$$

The value of the liquidity pool at any time $t \in [0,\infty)$*, is given by*

$$V(S_t; x_t^1, x_t^2) := x_t^1 \cdot S_t^1 + x_t^2 \cdot S_t^2\,.$$

Note that, under no arbitrage and no fee assumptions, it makes no difference whether the accounting is being done in S *or* P*.*

Using (2) we can show that

$$V(S_t; x_t^1) = \left(\frac{1-\theta}{\theta} + 1\right) S_t^1 x_t^1 = \frac{1}{\theta} S_t^1 x_t^1$$

or equivalently

$$V(S_t; x_t^2) = \left(\frac{\theta}{1-\theta} + 1\right) S_t^2 x_t^2 = \frac{1}{1-\theta} S_t^2 x_t^2\,.$$

From here we see that the value of the sub-pools with assets x^1 *and* x^2 *are* $\theta \cdot V(S_t)$ *and* $(1-\theta)V(S_t)$*, respectively.*

Next, we derive an alternative representation for V_t *that does not depend on* (x^1, x_t^2)*. To do that, note that*

$$1 = \frac{V(S_t; x_t^1, x_t^2)}{V(S_t; x_t^1, x_t^2)} = \frac{S_t^2 x_t^2}{1-\theta} \frac{\theta}{S_t^1 x_t^1}.$$

Hence

$$\begin{aligned} V(S_t; x_t^1, x_t^2) &= \left(\frac{S_t^1 x_t^1}{\theta}\right)^\theta \cdot \left(\frac{S_t^1 x_t^1}{\theta}\right)^{1-\theta} \\ &= \left(\frac{S_t^1 x_t^1}{\theta}\right)^\theta \cdot \left(\frac{S_t^1 x_t^1}{\theta}\right)^{1-\theta} \cdot \left(\frac{S_t^2 x_t^2}{1-\theta} \frac{\theta}{S_t^1 x_t^1}\right)^{1-\theta} \\ &= (x_t^1)^\theta (x_t^2)^{1-\theta} \left(\frac{S_t^1}{\theta}\right)^\theta \cdot \left(\frac{S_t^2}{1-\theta}\right)^{1-\theta}. \end{aligned}$$

Observe that

$$(x_t^1)^\theta (x_t^2)^{1-\theta} = (x_0^1)^\theta (x_0^2)^{1-\theta} \implies V(S_t) = (x_0^1)^\theta (x_0^2)^{1-\theta} \left(\frac{S_t^1}{\theta}\right)^\theta \cdot \left(\frac{S_t^2}{1-\theta}\right)^{1-\theta},$$

and

$$V(S_0) = (x_0^1)^\theta (x_0^2)^{1-\theta} \left(\frac{S_0^1}{\theta}\right)^\theta \cdot \left(\frac{S_0^2}{1-\theta}\right)^{1-\theta} \implies V(S_t) = V(S_0) \cdot \left(\frac{S_t^1}{S_0^1}\right)^\theta \cdot \left(\frac{S_t^2}{S_0^2}\right)^{1-\theta}.$$

In particular when asset S^2 is a numeraire and we set $\theta = \frac{1}{2}$ we have

$$V(S_t) = 2(x_0^1)^{1/2}(x_0^2)^{1/2}\sqrt{S_t^1} \quad \text{or} \quad V(S_t) = V(S_0) \cdot \left(\frac{S_t^1}{S_0^1}\right)^{1/2}.$$

The above example demonstrates two key properties of CFMs:

- A GMM 'automatically' rebalances liquidity pools so that the value of the pools with asset x^1 and x^2 is $\theta \cdot V(S_t)$ and $(1-\theta)V(S_t)$, respectively
- Providing liquidity to a CFM Ψ is equivalent to entering a long position on a perpetual derivative on the underlying asset with the payoff dictated by the value (in terms of price) V.

In Example 3.1 we have exploited a particular structure of GMM to derive the value of the LP portfolio in terms of asset prices (S^1, S^2) but not level of reserves. In turns out there is a generic way of doing that. We begin by observing that if Ψ defines a constant function market, then it is possible to apply the implicit function theorem to construct a convex function $\psi : \mathbb{R} \to \mathbb{R}$, called the *trading function*, such that

$$\Psi(x^1, x^2) = \kappa^2 \qquad \Leftrightarrow \qquad \psi(x^1) = x^2.$$

That is, ψ determines the amount of asset x we hold, when we have a given quantity y. We assume for simplicity that ψ is continuously differentiable. Given the reserves (x_t^1, x_t^2), an agent willing to sell Δx^1 to the pool will receive Δx^2 such that

$$x_t^2 - \Delta x^2 = \psi(x_t^1 + \Delta x^1) \implies \frac{\Delta x^2}{\Delta x^1} = -\frac{\psi(x_t^1 + \Delta x^1) - \psi(x_t^1)}{\Delta x^1}.$$

As in 1 the no-arbitrage condition (in the case of no fees ($\gamma = 1$)) implies that

$$-\partial_x \psi(x_t^1) = S_t.$$

As ψ is convex, we can take its Legendre transform, to define

$$\psi^*(s) = \sup_x \left\{ s \cdot x - \psi(x) \right\}.$$

Note that right hand side achieves its (unique) supremum when $\partial_x \psi(x) = s$. From 3.1 we see that for $s = -S_t$, we have $x = x_t^1$. Hence

$$-\psi^*(-S_t) = S_t \cdot x_t^1 + \psi(x_t^1) = S_t \cdot x_t^1 + x_t^2 = V(S_t)\,.$$

Hence the value of the LP position in the pool is $-\psi^*(-S_t)$ and can be readily computed using any off-the-shelf convex optimisation algorithm. As ψ^* is a convex function, we see that V is concave, and as ψ is decreasing, we see that V is increasing.

Example 3.2 (Uniswap V2)**.** *For a constant product market (as in Uniswap V2), $\Psi(x^1, x^2) = x^1 \cdot x^2 = \kappa^2$, so $\psi(x^1) = \kappa^2/x^1$. We compute $\psi^*(s) = -2\kappa\sqrt{-s}$, and hence $V(s) = 2\kappa\sqrt{s}$.*

Example 3.3 (Uniswap V3 - Concentrated Liquidity)**.** *In Uniswap V3, which essentially is as a collection of Uniswap V2 pools, liquidity providers specify the price range $[P^l, P^u)$, $P^l < P^u$ in which they wish to supply liquidity. For simplicity, assume that asset two is the numeriare and that level of reserves is (x_t^1, x_t^2). To facilitate the trade in the range $[P^l, P^u)$, Uniswap V3 introduces*

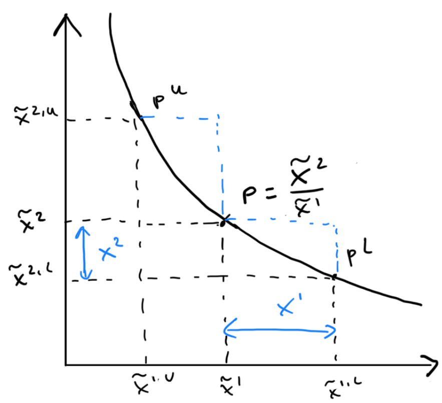


Figure 1: Inflated liquidity $(\tilde{x}^1, \tilde{x}^2)$ vs actual liquidity (x^1, x^2)

inflated (virtual) reserves $(\tilde{x}_t^1, \tilde{x}_t^2)$ such that

$$P_t = \frac{\tilde{x}_t^2}{\tilde{x}_t^1} \in [P^l, P^u) \quad \textit{and} \quad \tilde{x}_t^1 \tilde{x}_t^2 = \kappa^2\,.$$

This implies

$$\tilde{x}_t^1 = \frac{\kappa}{\sqrt{P_t}}\,, \ \textit{and} \ \ \tilde{x}_t^2 = \kappa\sqrt{P_t}$$

Now observe, see figure 1, that

$$x_t^1 = \tilde{x}_t^1 - \tilde{x}^{1,u} = \kappa\left(\frac{1}{\sqrt{P_t}} - \frac{1}{\sqrt{P^u}}\right), \quad \textit{where} \quad \tilde{x}_t^{1,u} = \frac{\kappa}{\sqrt{P^u}}$$
$$x_t^2 = \tilde{x}_t^2 - \tilde{x}^{2,l} = \kappa(\sqrt{P_t} - \sqrt{P^l})\,,, \quad \textit{where} \quad \tilde{x}^{2,l} = \kappa\sqrt{P^l})\,.$$

We also see that the constant market function can be re-written as

$$\Big(x_t^1 + \frac{\kappa}{\sqrt{P^u}}\Big)\Big(x_t^2 + \kappa\sqrt{P^l}\Big) = \kappa\,.$$

The value of the liquidity pool at any time $t \in [0, \infty)$, under no arbitrage condition 1 is given by

$$V(S_t; x_t^1, x_t^2) := x_t^1 S_t + x_2 = \kappa\Big(\frac{1}{\sqrt{S_t}} - \frac{1}{\sqrt{P^u}}\Big)S_t + \kappa\Big(\sqrt{S_t} - \sqrt{P^l}\Big)\,.$$

As observed in [2, 3], for any CFM V for any nonnegative, nondecreasing, concave, 1-homogenous[3] payoff function V, there exists a trading function Ψ, such that the value of the liquidity provision in CFM with with function Ψ matches this payoff function. In other words, a CFM with appropriately designed trading function Ψ dynamically adjusts the portfolio held by liquidity providers so that the value of this portfolio, at any time, is described by the payoff function V. This gives us two ways to understand a CFM:

- Through the defining trading function Ψ, indicating valid combinations of the two assets.
- Through the valuation function V, indicating the value of the CFM pool in terms of the prices of the two assets.

Concentrated liquidity, introduced in Uniswap V3, gives individual LPs control over what price ranges their capital is allocated to. This gives a practical way of creating liquidity provision that replicates a given function V, and has been observed and described in the Panoptic white paper [8].

3.2 Connection with perpetual options

Given a payoff function V, there is a simple connection between liquidity provision in a CFM and investment in a perpetual American option.

3.2.1 Perpetual option

We begin with a precise definition of a perpetual American contract with streaming premium. As we shall see, liquidity provision in a CFM is equivalent to entering a long perpetual derivative with the payoff dictated by the function V. This derivative is written on the underlying assets $S = (S_t)_{t\geq 0}$ traded in the pools.

Definition 3.4. *A perpetual contract with streaming premium, written on assets with price vector S, with payoff function $V : \mathbb{R}^n \to \mathbb{R}$, is a agreement between two parties, referred to as the long side and short side. The long side has the*

[3] That is, $\lambda V(S) = V(\lambda S)$ for all $\lambda > 0$ and S. Equivalently, we can assume that one of the assets in S_t is the numeraire asset, in which case the 1-homogeneity assumption is not needed.

right to terminate the contract at any time $t \geq 0$, at which point it will receive a payment of $V(S_t)$. In return, the long-side must pay to the short side $V(S_0)$ at the time $t = 0$ of inception as well as a continuous cash-flow of $(g_t)_{t \geq 0}$ per unit time, referred to as the streaming premium, up until the contract is terminated.

Remark 3.5. *While we here describe this transaction as involving an initial payment, it is possible to avoid this with appropriate margin accounting rules. Essentially, the long-side would post sufficient margin to account for the risk-adjusted expected appreciation of value of their position over time, and in addition to the streaming premium, their margin account value would be marked-to-market dynamically. For simplicity, we will focus on the initial payment formulation for now.*

3.2.2 Liquidity position in CFM and perpetual option

In order to highlight the connections between a CFM and a perpetual American option, we make the following observations. In a CFM, a liquidity provider (CFM-LP)

- initially deposits assets with value $V(S_0)$,
- receives fees at the rate f_t per unit time (which may vary). These fees may be withdrawn (under the Uniswap v3 protocol) at any time,
- at some future time τ (of the CFM-LPs choosing), may withdraw their assets, which will have value $V(S_\tau)$.

In a perpetual American option with streaming premium and payoff V, an investor purchasing the option

- initially purchases the option, for a cost $V(S_0)$,
- receives the streaming premium at a rate g_t per unit time (which may vary). This streaming premium may be positive or negative, and is received immediately,
- at some future time τ (of the investor's choosing), may exercise the option, which rewards them with value $V(S_\tau)$.

As we can see, assuming fees in the CFM are instantaneously predictable (i.e. the fee to be received from the CFM can be accurately estimated in advance), a no arbitrage argument suggests that we must have $f_t = g_t$, as otherwise one asset is strictly better than the other in every state of the world (over some short time horizon), which implies that an efficient market will focus all its trading in the better of these alternatives.

3.3 Panoptions within Uniswap

We can now give a stylized view of the Panoptic protocol, from the perspective of payoff functions. We have seen that a CFM corresponds to a (perpetual option with) concave increasing payoff function V, representing the value of the pool at a given price (if the CFM's assets were sold in the liquid market). We denote the payoff function associated to the Uniswap v3 pool (without panoptions) as V^U

The net position of the Panoptic protocol corresponds to the addition of liquidity to the Uniswap pool. This additional liquidity has value (at a given price S_t) given by a function $V^P(S_t)$, which is again a nonnegative concave increasing function. By working at the level of the payoff functions, we can see that the Uniswap pool, after the Panoptic protocol moves liquidity into the Uniswap pool, is described by the modified payoff function

$$V^{UP}(S_t) = V^U(S_t) + V^P(S_t).$$

Given the Panoptic protocol payoff function V^P must remain nonnegative concave increasing, there are limits to the trades that are possible. The requirement for nonnegativity corresponds to the assets provided from the Panoptic liquidity pool, which limits the scale of the payoff functions available.

Suppose a trader wishes to establish a position with (perpetual American) payoff $\eta(S_t)$; where we assume that η is 1-homogenous. This could be a call, put or other option. If the current state of V^P is given, the option η is available if (and only if) the function

$$V^P + \eta$$

is also a nonnegative concave increasing function.[4] If the trade η is accepted by the protocol, the position V^P updates to $V^P + \eta$, which alters the subsequently available trades.

The most basic trades in the Panoptic protocol (short-put positions, or equivalently, after the addition of capital, covered calls/cash secured puts) are those trades corresponding to[5] a concave increasing payoff η. These trades should be privileged within the protocol, as they are always available (provided sufficient liquidity is available from Panoptic LPs, which can be added to the position to ensure nonnegativity), and provide liquidity for further trading.

More general payoffs (calls, puts, etc...), which involve either decreasing or convex payoffs, will only be available when V^P is sufficiently large and concave to offset the desired exposures. Note that this implies that general payoffs can also involve the use of assets from the Panoptic liquidity pool – when accepting a trade η at time t, the assets committed from the liquidity pool move in

[4]Note this requirement limits the valid positions η to those which result in a *nonnegative* payoff. This is not so restrictive, as traders will effectively only be exposed to the returns on η, but limits the scale available, through implicit reference to size of the Panoptic liquidity pool.

[5]More precisely, any concave increasing payoff can be constructed as the sum of a collection of short-put positions and risk-free assets.

value from $V^P(S_t)$ to $V^P(S_t) + \eta(S_t)$, despite the Panoptic LP's market exposure remaining neutral (as the trader entering into the position η bears the corresponding market risk).

4 Arbitrage free streaming premia

In this section we will derive a no-arbitrage bound on f_t for the long position in a CFM, and then discuss how the presence of panoptions impact this calculation.

Consider a payoff function $V : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ which we assume to be twice differentiable. Let $(f_s)_{s\geq 0}$ denote the fee rate paid to the CFM-LP who holds the position with payoff V (this fee depends on the CFM fee and amount of assets traded in the CFM pools, at the current price level). Equivalently, we could consider the corresponding perpetual option investor, with a streaming premium $g_s = f_s$.

We consider a liquidity provider (LP) who deposits assets in a CFM. At time $t = 0$ the LP invests $V(S_0)$, by depositing assets into the liquidity pool. At any time t the LP can exit the pool receiving $V(S_t)$. Before they exit, they continuously receive fees at a rate f_t per unit time. (We assume that these can be obtained continuously through time, as would be the case in a Uniswap v3 market without fees.)

Natural questions arise:

- What are the risks of liquidity provision in a CFM?
- What should the fee rate $(f_t)_{t\geq 0}$ be, to prevent prevent arbitrage?

Remark 4.1. *We first study these questions using a classical continuous-time financial market with no frictions. This gives a reasonable base case for further development, as the no-arbitrage dynamics implied from the market without frictions will also typically prevent arbitrage in a market with frictions. (Generally speaking, adding frictions to a market increases the range of arbitrage free solutions.)*

4.1 Financial market model and assumptions

We make the following assumptions about the market:

- The agent can borrow and lend any amount of cash at the riskless rate.
- The agent can buy and sell any amount of assets x^1 and x^2.
- The above transactions do not incur any transaction costs and the size of a trade does not impact the prices of the traded assets.

We denote the risk free rate (which may be stochastic) by $(r_t)_{t\geq 0}$ and model the money market as

$$dB_t = r_t B_t dt\,, \qquad B_0 \geq 0\,.$$

We assume $r_t \geq 0$. We further denote the drift and diffusion coefficients (again, possibly stochastic) of the risky asset shadow prices by $\mu = (\mu_t^1, \mu_t^2) \in \mathbb{R}^2$ and

$$\sigma = (\sigma_t^{(1,1)}, \sigma_t^{(1,2)}, \sigma_t^{(2,1)}, \sigma_t^{(2,2)}) \in \mathbb{R}_+^4$$

respectively, and model the shadow price of the risky asset $(S_t^i)_{t\geq 0}$ by

$$dS_t^i = \mu_t^i S_t^i dt + \sum_{j=1}^{d} \sigma_t^{(i,j)} S_t^i dW_t, \quad S_o^i \geq 0\,, \tag{3}$$

where $W = (W_t^1, W_t^2)$ is a 2-dimensional Brownian motion. We assume that (μ, σ) are sufficiently regular that a (unique strong) solution to (3) exists.

Remark 4.2. *By allowing the drift and diffusion coefficients to be arbitrary processes we just saying that the prices are non-negative and continuous: our framework incorporates a rich family of common models such as Black–Scholes, Heston, SABR or local stochastic volatility models with possibly path dependent coefficients.*

4.1.1 Derivation of arbitrage-free fee rate bound

We will proceed in a similar way to the construction of the predictable loss in [5], and to classical arguments for no-arbitrage pricing in financial markets. Consider the wealth process Z of an agent who

- begins with zero capital[6],
- initially borrows a quantity $V(S_0)$ at the risk-free interest rate, which they use to establish a CFM position (which they can do by trading in the liquid market to obtain the desired risky assets for the pool),
- until time t, trades in the liquid market hold a quantity (Δ_s^i) of each asset at time s.

The dynamics of Z_t are given by

$$Z_t = \underbrace{V(S_t) - V(S_0)}_{\text{CFM gains}} + \underbrace{\int_0^t \sum_{i=1}^{2} \Delta_s^i dS_s^i}_{\text{trading gains}} + \underbrace{\int_0^t \Big(Z_s - \sum_{i=1}^{2} \Delta_s^i S_s^i\Big) r_s ds}_{\text{interest payments on uninvested wealth}} + \underbrace{\int_0^t f_s ds}_{\text{CFM fees}}.$$

We assumed V is smooth, hence we can apply Itô's lemma to obtain

$$\begin{aligned} Z_t = &\sum_{i=1}^{2} \int_0^t \Big[\partial_i V(S_s) + \Delta_s^i\Big] dS_s^i \\ &+ \frac{1}{2} \sum_{i,j=1}^{2} \int_0^t \partial_i \partial_j V(S_s) d\langle S^i, S^j\rangle_s + \int_0^t \Big[\Big(Z_s - \sum_{i=1}^{2} \Delta_s^i S_s^i\Big) r_s + f_s\Big] ds. \end{aligned}$$

[6] This is simply to avoid having to account for the interest they should earn on their initial capital

By setting $\Delta_s^i = -\partial_i V(S_s)$, we eliminate the first term, giving

$$Z_t = \frac{1}{2}\sum_{i,j=1}^{2}\int_0^t \partial_i\partial_j V(S_s)d\langle S^i, S^j\rangle_s + \int_0^t \Big[\Big(Z_s + \sum_{i=1}^{2}(\partial_i V(S_s))S_s^i\Big)r_s + f_s\Big]ds.$$

As the quadratic variation is $d\langle S^i, S^j\rangle_s = (\sigma_s\sigma_s^\top)^{(i,j)}S_s^i S_s^j ds$, this simplifies to

$$\frac{dZ_s}{ds} = \frac{1}{2}\sum_{i,j=1}^{2}\partial_i\partial_j V(S_s)(\sigma_s\sigma_s^\top)^{(i,j)}S_s^i S_s^j + \Big(Z_s + \sum_{i=1}^{2}(\partial_i V(S_s))S_s^i\Big)r_s + f_s.$$

If this quantity is positive, then we have a strategy which earns money with probability one over a short time period, that is, an arbitrage. Therefore, after rearrangement, we know that

$$f_s + Z_s r_s \le -\frac{1}{2}\sum_{i,j=1}^{2}\partial_i\partial_j V(S_s)(\sigma_s\sigma_s^\top)^{(i,j)}S_s^i S_s^j - \Big(\sum_{i=1}^{2}(\partial_i V(S_s))S_s^i\Big)r_s.$$

As Z is increasing in f (an agent who earns more fees will be wealthier) and we assume $r \ge 0$, there is a unique value of f such that above equation is an equality. Using this fee rate, we have $dZ_s/ds = 0$ and $Z_0 = 0$, and hence $Z_s = 0$. We denote this critical value[7]

$$\hat{f}_s = -\frac{1}{2}\sum_{i,j=1}^{2}\partial_i\partial_j V(S_s)(\sigma_s\sigma_s^\top)^{(i,j)}S_s^i S_s^j - \Big(\sum_{i=1}^{2}(\partial_i V(S_s))S_s^i\Big)r_s.$$

This critical fee rate can also be expressed in a model-free way, as

$$\hat{f}_t = -\frac{1}{2}\sum_{i,j=1}^{2}\partial_i\partial_j V(S_t)\frac{d\langle S^i, S^j\rangle_t}{dt} - \Big(\sum_{i=1}^{2}\partial_i V(S_s)S_t^i\Big)\frac{d\log B_t}{dt}.$$

Notice that the critical fee rate depends on the trading function via $\partial_i\partial_j V(S)$ (which is negative, as V is concave as long as the trading function Ψ is convex) and the quadratic variation of traded asset (which is implicitly related to the level of trading activity).

Note that this argument only uses a long-position in the CFM, and only establishes[8] the inequality bound $f_t \le \hat{f}_t$. If it were possible to perfectly short-sell the CFM (or the corresponding perpetual option), then a similar argument would yield the converse inequality. This may explain why the fee rate in Uniswap v3, is systematically below the critical fee rate, which is related to liquidity provision in CFMs yielding persistently poor returns.

[7] [5] give a similar calculation, to obtain a closely related quantity, which they call the permanent loss of the CFM.

[8] The presentation above assumes the agent starts at time 0, and derives the critical fee rate on this basis. To obtain the inequality bound $f_t \le \hat{f}_t$ for all times, we formally have to consider starting with zero capital at a time where the inequality is not satisfied, and showing that this gives a short-term arbitrage opportunity.

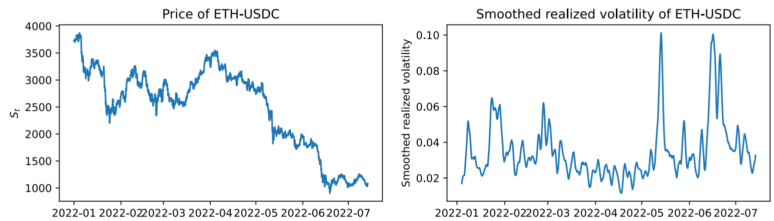


Figure 2: Price and smoothed realised volatility (100-day moving average of squared log-returns) in ETH-USDC market

Example 4.3. *For clarity of presentation we set the risk free interest rate* $r = 0$*, and assume that asset* S^2 *is a numeraire (hence the agent only invests in asset* $S = S^1$*). This means that*

$$V(S_t) = V(S_0)\left(\frac{S_t}{S_0}\right)^{\theta}.$$

The second derivative of V *is given by* $\partial_S^2 V(S_t) = \theta(\theta-1)\frac{V(S_0)}{S_0^2}\left(\frac{S_t}{S_0}\right)^{\theta-2}$*, which is negative since* $\theta \in (0,1)$*. The critical fee rate is then given by*

$$\begin{aligned}\hat{f}_t &= \frac{-1}{2}\theta(\theta-1)\frac{V(S_0)}{S_0^2}\left(\frac{S_t}{S_0}\right)^{\theta-2}\sigma_t^2 S_t^2 \\ &= \frac{\theta(1-\theta)}{2}\sigma_t^2 V(S_0)\left(\frac{S_t}{S_0}\right)^{\theta} = \frac{\theta(1-\theta)}{2}\sigma_t^2 V(S_t) \geq 0\,.\end{aligned}$$

As mentioned above, by analyzing the data in the Uniswap v3 pool (e.g. [5]) one can see that, in general, the fee rate is below critical fee rate. Nevertheless, since one cannot simply short position at the Uniswap (unless using an over-the-counter bespoke arrangement) it is not clear how one could realise a potential arbitrage opportunity when $f < \hat{f}$.

One possible counter argument is that, due to efficiency of markets, the fees f and critical fee rate $\hat{f}$ should remain aligned. This argument struggles for a few reasons, to do with the structure of the markets being considered.

- As the critical fee rate depends on the volatility of the asset, it will typically vary over short to moderate time periods. For example, we see in Figure 2 that the volatility of the market has varied significantly (e.g. by a factor of at 5) over the past year. However, the Uniswap fee rate is generally set over longer periods, and cannot easily equilibriate.

- As it is not possible to enter into a short position on Uniswap, there is currently only limited pressure on fees to equilibriate. This may be modified by trading in Panoptions, but would depend on the streaming premia which Panoptions yield. If short positions on Uniswap (corresponding to long-put positions in Panoptions) do not have sufficiently negative streaming premia, efficiency of markets would suggest that there will be significant demand for these products, which are only in limited supply (depending on the presence of long-options traders). If the streaming premia is endogenized (or tied to the economic fundamentals through the above no-arbitrage argument), then market efficiency in the Panoptions market would naturally impact the Uniswap fee rate, leading to equilibrium.

4.2 Bounds on Panoption streaming premia

Within Uniswap v3, it is only possible to enter into a long-liquidity provision position, corresponding to concave increasing payoff functions V. The introduction of Panoptions allows agents to effectively enter short positions relative to a Uniswap LP (in terms of the discussion in Section 3.3, the trader's payoff η can be non-concave and/or nonincreasing) and so the fee structure on Panoptions needs to be treated carefully.

Panoptions come with a streaming premium model, where the owner of a panoption with payoff η will receive a streaming premium with rate g (which depends on η and the current state of the market). By taking $g < 0$, we see that this can effectively model payment of a streaming premium fee.

By perfect analogy with the analysis in the previous section, we can write down the bound on the streaming premium fee which is required for no-arbitrage within the Panoption market (assuming trades can also be offset in the liquid market, as described before):

$$\begin{aligned} g_s \leq \hat{g}_t &= -\frac{1}{2}\sum_{i,j=1}^{2} \partial_i \partial_j \eta(S_t) \frac{d\langle S^i, S^j\rangle_t}{dt} - \Big(\sum_{i=1}^{2} \partial_i \eta(S_s) S_t^i\Big) \frac{d\log B_t}{dt} \\ &= -\frac{1}{2}\sum_{i,j=1}^{2} \partial_i \partial_j \eta(S_s)(\sigma_s \sigma_s^\top)^{(i,j)} S_s^i S_s^j - \Big(\sum_{i=1}^{2} (\partial_i \eta(S_s)) S_s^i\Big) r_s \,. \end{aligned}$$

Assuming the interest rate $r = 0$, we observe that for η convex, this quantity is negative, indicating that the owner of such a Panoption must be required to pay a substantial fee in order to avoid arbitrage opportunities.

We note in passing that the only quantity in this fomula which is not directly observable is the instantaneous volatility σ. Unlike in Black–Scholes pricing for traditional European options, where the volatility needs to be estimated over the remaining maturity period of the option, here we only need an estimate of the value of the instantaneous volatility at the present time.

5 Challenges for the Panoptic protocol

In this section we present some scenarios which, depending on the precise implementation of the Panoptic protocol, may render the market for Panoptions undesirable for participants. We will consider how various fees can be used to resolve these issues in later sections. This section should be interpreted as points for further discussion and development in the protocol specification.

In what follows, we say a position is a short-put position if it corresponds to a concave increasing payoff. These positions correspond to payoffs which can be achieved by trading within the Uniswap market. We describe the corresponding position (which is then available to be traded against general payoffs) as the short-put liquidity. A general payoff is one which is not concave increasing.

5.0.1 Streaming payments

Under the current specification of the protocol, a general position η can only be entered when a sufficient short-put position V^P is already in place (i.e. when $V^P + \eta$ remains concave increasing). Suppose an agent creates a short-put position V^P, then quickly afterwards offsets this by creating a corresponding position $\eta = -V^P$. They then close out the short-put position. If the protocol does not close their η position simultaneously, they have created a net invalid position for the Panoptic protocol. Note, this does not automatically mean that such a position is impossible, merely that the Panoptic protocol is unable to achieve the desired payoff by positioning in the Uniswap market.

Depending on the specification of the contract, the natural default counterparty is the Panoptic-LPs – the payoff η is guaranteed by the LP funds in the Panoptic liquidity pool. If the position η is not charged a sufficiently large streaming premium fee, this creates an arbitrage opportunity, which will be exploited against the Panoptic-LPs.

5.0.2 LP exposure to market risk

If it is possible to establish a net position which is not short-put (due to incomplete close-out of general payoffs when short-put liqudity is withdrawn) then, in addition to the above concerns regarding sufficient streaming premium fees, Panoptic-LPs are exposed to market risk (as the underwriting body for the general payoff), which should be made clear in the protocol rules.

5.0.3 Closeout and commission fees

Now suppose that general positions are closed out whenever short-put liquidity is withdrawn. The decision of which positions to close out needs to be made clear, and will have consequences for traders. For example, an agent establishing a general position will pay a commission fee to Panoptic-LPs (and possibly providers of short-put liquidity, depending on the protocol fee rules). Similarly, an agent establishing a short-put position will pay a commission fee to Panoptic-LPs.

However, this creates an opportunity for a Panoptic LP, who can withdraw liquidity immediately after the commission fee is received, and close out the downstream positions. If the Panoptic-LP is not charged a withdrawal fee (and similarly if a short-put position is not charged an execution fee), and the fee for providing new Panoptic-LP liquidity is small, this presents an arbitrage opportunity.

This suggests that some close-out/withdrawal fees for short-put positions and Panoptic-LPs, or an escrow system for the commissioning fee income (where fees are only paid when liquidity is provided for a sufficient period of time), should be in place.

5.0.4 LP rewards for long-term positions

In the current specification, the Panoptic-LP only receives fees when option positions are created. This leads to a difficulty when a short-put position is held over a long period, as the short-put holder is exploiting Panoptic-LP liquidity without rewarding Panoptic-LPs for this. In particular, this means that transferring a short-put position between third parties (through a further smart contract) is strictly better than establishing them directly within the protocol, as no fees would need to be paid.

5.0.5 Margin rules and closeout

The traditional view of margin accounts is that they eliminate counterparty risk, as any party which defaults on a contract will provide sufficient cash to enable the party defaulted on to recreate their position in the market. If a rule is adopted whereby the exercise of a short-put position will cause the closeout of a general position η, it becomes difficult to determine the risk-management benefits of the margin account correctly – at the moment of closeout (which constitutes a form of default from the perspective of the holder of η), there is no liquidity available with which they can reestablish their position. This emphasises the necessity of compensating these agents when passively closed out.

5.0.6 Closeout risk

A higher order effect, depending on the implementation of closeout rules, is as follows: Suppose the protocol has established a large short-put position, but has almost fully offset this through the sale of general positions. The available short-put liquidity is then quite small. If an agent purchases a general position, they will increase the risk of closeout for all general positions (depending on the priority rule for closeout). This should be reflected in the fees paid for establishing a general position, and in the priority rules for closeout.

5.0.7 Uniswap and Panoptic fees for Liquidity

Under the current specifications, a short-put position will earn, in addition to the changes in value due to the position, the same fees as in Uniswap v3. This implies that Panoptions will not directly contribute to closing the statistical arbitrage that is currently present in Uniswap, and there is limited direct incentive for a short-put trader to trade through Panoptions, rather than as a Uniswap-LP. (There is, however, an indirect incentive, as Panoptions enable leveraged positions.) This may limit the attractiveness long-term of the long-put position, if it remains a net-loss position for traders.

6 Fees and streaming payments

We propose below a fee structure extending that already proposed as part of the panoptic protocol, which ensures the following properties:

- Short-put positions are rewarded at a rate at least as high as a direct deposit in Uniswap.
- General positions are required to pay a no-arbitrage streaming premium fee, or Uniswap fees if higher.
- Panoptic-LPs are protected from arbitrage in case of short-put exercise, if there is an incomplete closure of corresponding short-put positions (they face a market risk, but are rewarded at a higher rate than uniswap fees). They receive fees depending on utilization of the pool, rather than purely through churning of option positions.

These three properties should ensure that the Panoptic protocol provides economic value to all participants – Panoptic-LPs earn commission fees and ongoing fees when their liquidity is used; short-put positions earn more than they would earn in Uniswap, and general positions are made available for use, at an economic fee.

6.1 Proposed fee structures

We here outline some proposed cashflows for each agent type, which should eliminate the concerning situations above. The precise details of how these payments are made (in particular, who pays the gas fees and initiates the corresponding balancing transactions between margin accounts) we leave for future discussion, as these depend on the precise structuring of the panoptic protocol. These suggestions should only be taken as indicative of the broad economic considerations at play, rather than as a concrete or exhaustive suggestion.

6.1.1 Panoptic short-put holder

An options trader who '**purchases a short-put**' effectively is moving assets from the panoptic pool into Uniswap. Their cashflows at each time are the sum of:

- At initial time

SA1: Depositing a sufficient margin account balance.

SA2: Payment of initial commission fees based on pool utilization.

- During the life of the option

SB1: Payment of leverage fees (given by a fixed percentage of the initial commission fee SA2 if the trade were to be restarted at the current time)

SB2: Margin account rebalancing payments (based on price moves in Uniswap)

SB3: Receipt of share of Panoptic streaming premium

- At the time of exit:

SC1: The payoff in the position in Uniswap, including the fees earned in Uniswap, minus the initial value deposited into Uniswap

SC2: Receipt of remaining margin account value

SC3: Liquidation fees to general position holders (GC3), if needed

SC4: Receipt of partial refund of commission fees if closeout occurs because of the departure of a liquidity provider (LC2)

6.1.2 Panoptic general position holder

An options trader who enters into a general (not concave increasing) position effectively is moving assets around and out of Uniswap into the Panoptic pool. Their cashflows at each time are the sum of:

- At initial time

GA1: Depositing a sufficient margin account balance.

GA2: Payment of initial commission fees, based on size of short-put position and LP pool at relevant level.

- At each time before exit:

GB1: Payment of leverage fees for impact on Panoptic LPs (given by a fixed percentage of the corresponding initial commission fee GA2 if the trade were to be restarted at the current time).

GB2: Margin account rebalancing payments (based on price moves in Uniswap)

GB3: Payment of share of Panoptic streaming premium (cf. SB3)

- At the time of exit:

GC1: The change, over the lifetime of the option, of their uniswap position, including fees earned.

GC2: Receipt of remaining margin account value.

GC3: Receipt of partial refund of commission fees if closeout occurs because of the departure of a short-put or liquidity provider (SC3, LC2)

6.1.3 Panoptic Liquidity Provider

A **Panoptic liquidity provider** faces the following cashflows:

- When they first provide liquidity

  LA1: Assets deposited in Panoptic liquidity pool

- At each time:

  LB1: Receipt of initial (SA2, GA2) commission fee and leverage (SB1) fees

  LB2: Receipt of any net imbalance in streaming premia (GB2−SB3)

  LB3: Any changes in value due to an excess of short positions

- At time when they withdraw liquidity

  LC1: Receive value of share of Panoptic pool

  LC2: Liquidation fees if needed (SC3, GC3)

6.2 Fee specifications

We now specify the various components of these fees.

Margins: (SA1, LA1, SB2, GB2, SC2, GC2) These are determined through margin premia calculations and mark-to-market conventions.

SA2: This is specified on the Panoptic protocol webpage. The use of a decreasing commission based on pool utilization helps ensure liquidity providers achieve a desired revenue stream.

GA2: This is paid to enter a general position, and would naturally be based on the volume of short-put liquidity available. The role of these payments depends on the details of the protocol – if the protocol automatically shuts down general positions when liquidity falls to zero, then payments GA2 can be held in escrow to provide a reward to traders for the adverse selection effect of their positions being closed. In particular, if there is little remaining short-put liquidity available (due to many general positions already having been created), creation of a new general position would increase the risk of adverse closeout to all general positions, and these fees could be held to compensate this risk.

SB1, GB1: This payment is needed to compensate liquidity providers for option positions which are held over long durations. A natural rule of thumb is to set this at a small percentage (say 5%) of the initial commission rate SA2 or GA2, perhaps with a rule that this is only implemented for positions which are held for longer than 20 days.

SC3, GC3: This payment is a premium paid when a short-put position is closed, to compensate general positions which may need to be closed out. A natural requirement is for some percentage of the commission fee GA2 to be refunded to the short-put holder as payment GC3.

SP — Streaming Premia: (SB3, GB3, LB2) This payment is used to ensure that purchasing short-put Panoptions is preferable to directly trading in Uniswap, and to remove the arbitrage opportunities for general positions. Essentially, there are two components to this: (1) general holders remove liquidity from uniswap, and therefore should compensate for the corresponding uniswap fee income (f_t). (2) short-put holders are entering into positions with a negative no-arbitrage streaming premium $\hat{g}$, and should compensate other market participants for this.

General position holders therefore pay a streaming premium fee $\texttt{GB3} = \max\{f_t, -\hat{g}_t\}$, where typically $-\hat{g}_t > f_t$. Short-put holders are guaranteed to receive the Uniswap fees f_t, together with the streaming premium $\texttt{SB3} = \max\{0, -\hat{g}_t - f_t\}$. This is shared proportionally among short-put holders, in proportion to the utilization of their contribution by general option holders.

If there is an excess of general positions (due to incomplete closeout on withdrawal of short-put liquidity), the residual streaming premium payments revert to the Panoptic LPs (as payment LB2).

Note that this ensures Panoptic-LPs and long-put positions are rewarded for the presence of general position traders. In any event, Panoptic-LPs achieve a fully-funded position (as LB2 implies they recieve the streaming premium for their implied market exposure due to an excess of short-put positions). Short-put positions are rewarded with a higher cashflow than they would achieve from Uniswap fees, partially eliminating the systematic losses of liquidity provision. General option positions must pay a larger streaming premium fee than the negative of the uniswap fees, but are rewarded with access to a short position, which is not otherwise available on the market.

SC1: This is simply the final uniswap value, including fees. We note that these fees may come in part from streaming premium payments made by general option positions, as given in the difference between GB3 and SB3 (i.e. when a general option holder alters the position of a short-put holder, the streaming premium they pay will be partly used to compensate the short-put holder for the corresponding Uniswap fees).

GC1: This is the change in the uniswap pool value. Some fees may be included in this, if the general position includes some concave increasing parts, which would earn uniswap fees. (However, these could be excluded for simplicity, as the higher fees on general panoptions should make it preferable to provide short-put liquidity as part of a portfolio, rather than including it in a general position.)

LB3: We expect that the implementation of the protocol will include a provision by which, when short-put positions are closed, if there is insufficient short-put liquidity remaining, some general option positions will be closed out. However, depending on details of implementation, this may not occur completely, resulting in a net not-short-put position being created. In this case, LB3 accounts for the changes in the value of the Panoptic pool which result from this implied market exposure (and LB2 ensures that Liquidity Providers are compensated for this risk).

LC2: When a Panoptic-LP decides to withdraw liquidity, they may need to compensate option holders for the consequence loss of liquidity within the pool. One approach would be to use a process similar to that used for the commission fees (SA2, GA2), to issue (partial) refunds of the commission fees to option holders whose positions are closed out passively by a Panoptic-LP departure. The closeout process needs to be specified fairly carefully, particularly if the total volume of options is permitted to exceed the LPs position (through offsetting effects of long and short option positions).

6.2.1 Remaining issues

One point which is not fully developed above, but will be important in application, is the sizes and flows of the payments associated with closeout of a position, particularly when the closeout is initiated by another party to the one being closed out. This can occur either when a liquidity provider withdraws from the panoptic pool (which can closeout short-put holders and general option holders), or when a short-put holder exits their position (which can closeout general option holders, depending on the rules of the protocol). Some exit fees, to compensate for this adverse selection effect[9], would be natural to implement.

Note also that the use of a margin requirement prevents agents from simply allowing a position to continue indefinitely, as they will be required to continue to service margin calls, leverage fees and streaming premia. The precise details of the margin calculations, mark-to-market rules, and related issues, are beyond the scope of this report.

7 Initial and maintenance margin

Allowing certain parties to open leveraged long and short positions on Uniswap (or other CFMs) using the Panoptic protocol opens the Panoptic liquidity providers to credit and liquidity risks.

7.1 Credit, liquidity risk and funding

Credit risk arises from the situation where the party that opened the leveraged or short position on Panoptic but the collateral balance controlled by the Panoptic protocol falls below the value of their position. At that point a rational party

[9] The usual argument that the margin payments fully compensate for adverse selection does not hold here, as these closeouts correspond to being forced out of positions which are subsequently not available to re-enter. This leaves market participants open to adverse selection risks, where their exposures are changed at times they would not choose.

will walk away from such position opening the panoptic liquidity providers to a loss. This risk can be managed and mitigated by Panoptic protocol holding sufficient collateral, which will be defined below, and allowing any participant in the Ethereum blockchain to take over delinquent parties' position while it is still profitable (execute a close-out transaction on the Panoptic protocol to close the offending party out and take over).

Liquidity risk arises when no participant on the Ethereum network is willing to execute the closeout transaction. This may arise at times of high uncertainty or other market dislocations. It can fundamentally only be mitigated by the collateral requirements being sufficiently large to make closeout attractive in most situations.

Fix a period of time $\tau > 0$ e.g. 1 hour or 24 hours. The time must be sufficiently long for all the participants in the blockchain to be able to observe state and execute transactions.

We will use S_t to denote the underlying price; in previous sections this denoted not the Uniswap v3 pool implied price but a price on the external exchange. Due to presence of arbitraguers we know that the Uniswap implied price tracks the external price closely. Thus whenever a value like S_t is needed as an input to a calculation we assume that Panoptic protocol contract can read it on chain from the Uniswap v3 contract and this is sufficiently close to the "true" price.

Let us first focus on the credit risk: the change in value of a position of the party between time t and $t+\tau$ will be $\eta(S_{t+\tau})-\eta(S_t)$. Initially we may think that it is enough for the margin to hold balance x such that $\mathbb{P}(\eta(S_{t+\tau})-\eta(S_t)+x \leq 0) \leq \alpha$ for some small probability α. This would lead us to set the credit risk component of the margin to

$$\mathrm{VaR}_\alpha(\eta(S_{t+\tau})-\eta(S_t)) = \inf\{x \in \mathbb{R} : \mathbb{P}(\eta(S_{t+\tau})-\eta(S_t)+x \leq 0) \leq \alpha\}\,.$$

However this doesn't tell us how big the shortfall would be in case the collateral reserve x is not sufficient. For this reason it makes more sense to set the value at the expected shortfall in case the reserve from VaR is insufficient:

$$\begin{aligned}
&\mathrm{ES}_\lambda(\eta(S_{t+\tau})-\eta(S_t))\\
&= \frac{1}{\lambda}\int_0^\lambda \mathrm{VaR}_\alpha(\eta(S_{t+\tau})-\eta(S_t))\,d\alpha\\
&= \mathbb{E}[-\eta(S_{t+\tau})-\eta(S_t)|\eta(S_{t+\tau})-\eta(S_t) < \mathrm{VaR}_\lambda(\eta(S_{t+\tau})-\eta(S_t))]\,.
\end{aligned}$$

However, in the event of a potential default (where the owner of an option no longer holds sufficient margin, and so their position is due to be closed), the margin account is designed to enable counterparties to close the position, over a reasonable timeframe, without incurring a loss. This means that we should also include the streaming premium which the position is required to pay (during the close-out period) in the computation. As the streaming premium is random (as it depends on the current price), we should include it in the risk calculation.

Here we arrive at a distinction between positions which provide liquidity to uniswap and those which do not. For a position which removes liquidity from uniswap, the panoption streaming premium is always required to be paid, and so should be included directly. However, for a position which adds liquidity to uniswap (i.e. a short-put position and compound positions based on it), there is no guarantee that the streaming premium will be received, as this depends on the utilization of the panoption component of the uniswap pool by other panoption purchasers. Recalling that, for a panoption holder, $\hat{g} > 0$ indicates receiving a fee, while $\hat{g} < 0$ indicates payment of a fee, if we assume a utilization level $\rho < 1$ (which may be $\rho = 0$ for simplicity) the ultimate streaming premium payments occur at the rate

$$\hat{g}_t^* = \rho \max\{\hat{g}_t, 0\} + \min\{\hat{g}_t, 0\} = \rho[\hat{g}_t]^+ - [\hat{g}_t]^-, \tag{4}$$

with $[\cdot]^+$ and $[\cdot]^-$ the positive and negative part operators respectively.

This can be further corrected to account for a liquidity providing position earning the uniswap fees, if desired. The effect of including $\rho < 1$ is to reduce the implied streaming premium which is earned by providing liquidity through panoptions.

Thus we arrive at the minimum value of collateral which a party must hold which covers the credit risk and the streaming fee payment:

$$m_t = \mathrm{ES}_\lambda \left[\eta(S_{t+\tau}) - \eta(S_t) - \int_t^{t+\tau} \hat{g}_s^* \, ds \middle| \mathcal{F}_t \right] .$$

If the collateral balance falls below this it presents too much credit risk to Panoptic liquidity providers and hence the protocol will allow the position to be adversarially closed out.

The protocol should set an initial margin level by scaling the maintenance margin by a factor:

$$m_t^{\text{initial}} := (1 + c) m_t \,,$$

with $c > 0$ to prevent parties from immediately being open to a close-out trade due to market price moves.

7.2 Stochastic models and linearisation

The formula for maintenance margin (7.1) is purely abstract: one needs to choose a stochastic model to be able to evaluate this. The model can be any of Black–Scholes, Heston, SABR, ... Even upon a choice of model and calibration of model parameters it will be impossible to evaluate (7.1) on the Ethereum blockchain: the gas cost itself would be prohibitive. However, being able to know (7.1) on chain is essential because the contract has to decide whether a close-out trade is permitted or not.

This is where being able to effectively simplify (7.1) is essential. This can be done by observing that most of the variability in m_t will come from the moves in price of the underlying assets (which can be observed from relevant Uniswap pools) and perhaps few other variables which can be observed or easily calculated on-chain e.g. current volatility estimate from the relevant Uniswap pool trades.

To be more specific let us assume that the underlying Uniswap pool sets price $\frac{P_t^{1,CFM}}{P_t^{2,CFM}}$ at time t for an infinitesimally small trade. We are implicitly assuming that one of the assets in the pool is the numeraire asset and so $\frac{P_t^{1,CFM}}{P_t^{2,CFM}}$ is the price of the other (implicitly risky asset) in terms of the numeraire (e.g. USDC is the numeraire, ETH is the risky asset). Let us assume that we wish to use the Black–Scholes model for calculating the maintenance margin. Then we postulate that $S_t \approx \frac{P_t^{1,CFM}}{P_t^{2,CFM}}$ and its evolution in the real world measure is

$$dS_t = \mu S_t \, dt + \sigma S_t \, dW_t^{\mathbb{P}} \,,$$

where μ and σ are model parameters to be chosen (e.g. by calibrating to historical data or forward looking data from options markets).

The credit risk part is $\mathrm{ES}_\lambda[V(S_{t+\tau}) - V(S_t)|S_t]$. Given that we know that $S_{t+\tau}|S_t$ has log-normal density with mean and variance fixed by μ, σ we can easily obtain $\mathrm{ES}_\lambda[X_{t+\tau}|S_t]$ by numerical integration or by a Monte-Carlo approximation. Such computation will of course be too hefty for Ethereum and so instead we postulate that

$$\mathrm{ES}_\lambda[V(S_{t+\tau}) - V(S_t)|S_t] \approx \sum_{i=0}^{n} \mathrm{RF}_\eta^{(i)} S_t^i \,,$$

where $\mathrm{RF}^{(i)} = \mathrm{RF}_V^{(i)}(\lambda, \mu, \sigma, \tau)$ is the i-th order risk factor which can easily be obtained off-chain. On-chain we will only need to evaluate calculation involving RF_V^i (which Panoptic protocol can store and update via governance), S_t (which can be read from the underlying Uniswap pool) and a sum and products.

Similarly we can set

$$\begin{aligned} \mathrm{ES}_\lambda \left[-\int_t^{t+\tau} \hat{g}_u^* \, du \middle| S_t \right] &= \frac{1}{2} \mathrm{ES}_\lambda \left[\int_t^{t+\tau} \left(\rho[\eta''(S_u)]^+ - [\eta''(S_u)]^- \right) \sigma^2 S_u^2 \, du \middle| S_t \right] \\ &\approx \sum_{i=0}^{n'} \mathrm{FF}_\eta^{(i)} S_t^i . \end{aligned}$$

Here $\mathrm{FF}_\eta^{(i)} = \mathrm{FF}_\eta^{(i)}(\lambda, \mu, \sigma, \tau)$ are the fee factors. The appropriate risk and fee factors are obtained by ordinary least squares for representative values of S_t and for the expectations evaluated by Monte–Carlo or numerical integration. This is an off-chain computation.

Note that if we chose another model for the underlying e.g. Heston or SABR the only thing that would change is that we now need two "observable" values on chain: the current risky asset price S_t in terms of numeraire and its volatility. We will then have

$$\mathrm{ES}_\lambda[\eta(S_{t+\tau}) - \eta(S_t)|S_t, \sigma_t] \approx \sum_{i,j=0}^{n} \mathrm{RF}_\eta^{(i,j)} \sigma_t^i S_t^j$$

and

$$\mathrm{ES}_\lambda\left[-\int_t^{t+\tau}\hat{g}_u^*\,du\,\middle|\,S_t,\sigma_t\right]\approx\sum_{i,j=0}^{n'}\mathrm{FF}_\eta^{(i,j)}\sigma_t^i S_t^j\,.$$

Note that:

i) This only sets out a high level overview how we could go about providing margin levels and fee premiums in a principled way on the Panoptic protocol. The next step to move further would be to decide which stochastic model we favour and carry out analysis of how many factors are needed and their stability. As we're looking at basically a Taylor series expansion including more terms will improve stability of the risk factors but it means more data needs storing on Ethereum.

ii) Using $S_0 - S_t$ (and $\sigma_0 - \sigma_t$ in case of stoch. vol. model) with some reference values S_0 and σ_0 will improve stability but again, it's another value to store.

iii) As currently written we need risk factors for each payoff V. This means that further approximation / simplification will be needed to split it into a part that's position dependent and a part that's universal, see Example 4.3.

Remark 7.1. *The rule here, based on expected shortfall, is often simplified to give an initial and maintenance margin based on a multiple of the asset values held. This is the motivation behind 'haircut' rules in traditional markets. While simple, this poses a difficulty for the panoptic protocol, as panoptions do not have a simple 'price' (as they are only bought on margin, so only the changes in their prices are seen). The natural analogue to the 'price' of a perpetual option is the streaming premium (which is chosen to force the price to zero), which is model-based.*

The current specification of the panoptic protocol uses instead a rule based on the 'notional value' of the assets. This needs to be carefully specified, particularly for compound options (such as strangles or butterflies), as the 'notional value' of the margined position is not simple. Given the substantially higher observed volatilities in crypto assets, it is not clear that rules of thumb from classical markets are appropriate.

8 Agent-based model - Panoptic and Uniswap v3

We turn into numerical simulations to study the Streaming Premia SB3, GB3 in different scenarios of low / high price volatility and low / high CFM fee $(1-\gamma)$, as well as the initial margin account required from the short-put holder and the general position holder (denoted GPH). In this section we model a CFM with concentrated liquidity like Uniswap v3. We repeat the simulations for a CFM without concentrated liquidity like Uniswap v2 in Appendix B.

We consider three agents, a) a short-put holder who moves assets from the Panoptic pool into Uniswap, b) a general position holder who utilises a portion of the short-put holder assets by moving them back in the Panoptic pool, and pays a streaming premia to compensate for the Uniswap fee income the short-put-holder would receive had they not entered the general position. And c) an arbitrageur who trades between the CFM and a reference market to make risk-free profits.

8.0.1 Short-put holder and CFM

We consider a CFM as in Example 3.3 with $\theta = 1/2$, x^1 the numeraire. Initial reserves (x_0^1, x_0^2) are entirely provided by a short-put holder through the Panoptic pool.

8.0.2 General position holder

At initial time a general position holder (denoted by GPH) withdraws a portion $\beta \in (0, 1]$ of the CFM liquidity in a certain tick range and deposits it back in the Panoptic pool. The reserves left in that tick range satisfy the trading equation 3.3 with the constant $\kappa \leftarrow (1-\beta)\cdot\kappa$.

When the position is closed at time τ, the GPH gets a payoff

$$\begin{aligned}\eta_{S_0}(S_\tau) = \beta\cdot\kappa\Big[\Big(\frac{1}{\sqrt{S_0}} - \frac{1}{\sqrt{P^u}}\Big)S_0 + (\sqrt{S_0} - \sqrt{P^l})\Big] \\ - \beta\cdot\kappa\Big[\Big(\frac{1}{\sqrt{S_\tau}} - \frac{1}{\sqrt{P^u}}\Big)S_\tau + (\sqrt{S_\tau} - \sqrt{P^l})\Big],\end{aligned}$$

which is a convex function in S_τ, see Figure 3.

In other words, the GPH has purchased a perpetual American option with payoff $\eta_{S_{t_k}}(\cdot)$. The theoretical non-arbitrage streaming fee using a delta hedge is

$$\hat{g}_t(S_t) = -\frac{1}{2}\sum_{i,j=1}^{2}\partial_i\partial_j\eta_{S_0}(S_t)\frac{d\langle S^i, S^j\rangle_t}{dt} - \Big(\sum_{i=1}^{2}\partial_i\eta_{S_0}(S_t)S_t^i\Big)\frac{d\log B_t}{dt}.$$

Under zero risk-free rate, $\hat{g}_t$ is negative because of the convexity of $\eta_{S_{t_0}}(\cdot)$. We can see $\hat{g}_t$ as the rate fee paid by the GPH to the short-put holder, the issuer of the perpetual American option. The shaded area in Figure 3 corresponds to the tick range where the GPH opened their position, which in addition is the only price range where $\hat{g}_t$ is non-zero.

Remark 8.1. *We need to be a little careful here, as the payoff should always be shifted (vertically) to ensure the current value of the payoff is zero – in this case, as $S_0 = 10.5$, this is already the case. This shift is needed to account for the fact a GPH does not pay initially when purchasing a panoption.*

8.0.3 Arbitrageur

Price discovery occurs in a second reference market, where S_t denotes the price of asset x^2 in terms of the numeraire x^1 at time t. We model the price S_t by a

Geometric Brownian Motion

$$dS_t = \sigma S_t dW_t, \quad S_0 > 0.$$

Whenever the price of asset x^2 in terms of the numeraire x^1 in the CFM is different than S_t, there might be an arbitrage opportunity. An arbitrageur will automatically make the necessary trades to maximise their profit. The arbitrageur who trades $(\Delta x^1, \Delta x^2)$ when reserves are (x^1, x^2) will incur cost $f(\gamma, x^1, x^2, \Delta x^1, \Delta x^2)$ and solves the optimisation problem

$$\Delta x^{2,*} := \max(\Delta x^2_{\text{CFM, CEX}}, \Delta x^2_{\text{CEX, CFM}}),$$

where

$$\begin{aligned}\Delta x^2_{\text{CFM, CEX}} := \arg\max_{\Delta x^2} \Delta x^1 \cdot S^1 - \Delta x^2 - f(\gamma, , x^1, x^2, \Delta x^1, \Delta x^2) \\ \text{such that } \Psi(x_1 - \Delta x^1, x_2 + \Delta x^2) = \Psi(x_1, x_2), \quad \Delta x^2 \geq 0, \\ \Delta x^1 \cdot S^1 - \Delta x^2 - f(\gamma, x^1, x^2, \Delta x^1, \Delta x^2) \geq 0\end{aligned}$$

and

$$\begin{aligned}\Delta x^2_{\text{CEX, CFM}} := \arg\max_{\Delta x^2} \Delta x^2 - \Delta x^1 S^1 - f(\gamma, x^1, x^2, \Delta x^1, \Delta x^2) \\ \text{such that } \Psi(x_1 + \Delta x^1, x_2 - \Delta x^2) = \Psi(x_1, x_2), \quad \Delta x^2 \geq 0, \\ \Delta x^2 - \Delta x^1 S^1 - f(\gamma, x^1, x^2, \Delta x^1, \Delta x^2) \geq 0\end{aligned}$$

with the subindexes of $\Delta x^2_{\cdot,\cdot}$ denoting the order of the markets where the arbitrageur goes short and long in asset x^2. If $\Delta x^{2,*} = 0$, then there is no possible trade for which an arbitrageur makes a profit.

8.0.4 Fee specifications of the simulation

Following the fee specifications in Section 6.1 and focusing in SB3, GB3, the agents will receive / pay the following:

a) **Short-put holder**:

- At $t = 0$ deposits sufficient liquidity in a margin account balance.
- For $t \in [0, T)$ receipt of CFM fee income given by $(1 - \gamma)$ paid by the arbitrageur's trades.
- For $t \in [0, T]$ receipt of the Panoption streaming premia paid by the GPH (denoted by SB3 in Section 6.1).

b) **General position holder**

- At $t = 0$ deposits sufficient liquidity in a margin account balance.
- For $t \in [0, T]$ payment of the Panoption streaming premia to the short-put holder (denoted by GB3 in Section 6.1).

c) **Arbitrageur**

- For $t \in [0, T]$, payment of the CFM fee to the short-put holder.

8.1 Simulation

We run the simulation on discrete times $\Pi := \{0, t_1, t_2, ..., T\}$ with initial reserves (x_0^1, x_0^2) and fee $(1-\gamma)$; initial price S_0 and volatility σ.

For every $t \in \Pi$:

1. If $t = 0$, the short-put holder moves the liquidity from the Panoptic pool to the CFM.
2. The arbitrageur makes the necessary trades resulting from optimisations in the CFM and in the reference market to make a risk-free profit, and pays the corresponding CFM fee to the short-put holder.
3. If $t = 0$, a GPH opens a position by withdrawing liquidity from the CFM. We run three scenarios displaying different payoffs.

We study the initial margin account for the GPH and short-put holder, corresponding to positions with convex and concave payoff respectively:

- Positions with convex payoff – positions that withdraw liquidity from Uniswap to deposit it in the Panoptic pool. These correspond to Figures 3a, 3b, 3c.
- Positions with concave increasing payoff – positions that purely move liquidity from the Panoptic pool to Uniswap. These correspond to Figure 3d. Note that the income fee for these positions will come from a GPH that utilises a portion of the assets to open their position. For the fee payments in these positions, we assume a utilization rate $\rho = 0.5$ in equation (4).
- Positions with general payoff – positions that both withdraw and deposit liquidity in Uniswap from the Panoptic pool. These correspond to Figures 3e, 3f, which are not concave increasing. We again consider the utilization rate $\rho = 0.5$ in equation (4).

More details about the different positions that we study can be found in Appendix A.

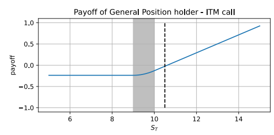
We calculate the initial collateral the position holder must deposit in the margin account, given by

$$m_0 = \mathrm{ES}_\lambda \left[\eta(S_1) - \eta(S_0) - \int_0^T \hat{g}_s^* \, ds \middle| S_0 = s_0 \right] , \tag{5}$$

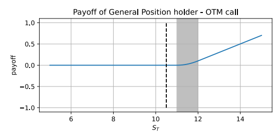
where $\eta(S_t)$ is the value/payoff of the option at time t.

The aim for our simulations is two-fold, a) study the the fee income of a short-put holder with and without the Panoption fee income (SB3), and b) study the the initial margin account (5) in different payoff scenarios.

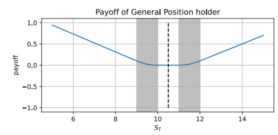
We take $\sigma \in \{0.2, 0.4, 0.6, 0.8\}$, fix $S_0 = 10.5, x_0^1 = 100, x_0^2 = 10$ and run the above simulation 1000 times for different values of $\gamma \in [0.97, 1]$.



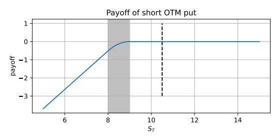
(a) Payoff ITM call.



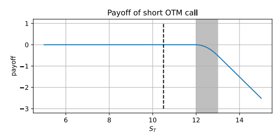
(b) Payoff OTM call.



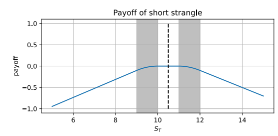
(c) Payoff strangle option



(d) Payoff short OTM put



(e) Payoff short OTM call



(f) Payoff short strangle

Figure 3: Payoff of positions. Shaded area indicates the tick from where the GPH withdraws liquidity. The dashed line indicates the initial price of the risky asset in the simulation.

8.1.1 Initial margin account of general position holders - positions with convex payoff

We consider in this subsection the initial margin account of the positions that withdraw liquidity from Uniswap to deposit it in the Panoptic pool. These correspond to Figures 3a, 3b and 3c. In this case, as the payoff is convex, we have $\hat{g} = \hat{g}^* < 0$ in equation (4).

Figure 4 shows the collateral needed by the GPH to open their position. The expected shortfall is calculated with $\lambda = 0.9$ and fixed Uniswap fee 0.3%. In addition, we also plot $\mathrm{ES}_\lambda[\eta(S_1) - \eta(S_0)]$, $\mathrm{ES}_\lambda[-\int_0^1 \hat{g}_s ds]$ and their sum, i.e. the expected shortfall when considering the payoff and streaming premium separately. Recall that $\hat{g}_s$ is negative because of the convexity of $\eta(S_t)$, hence $\mathrm{ES}_\lambda[-\int_0^1 \hat{g}_s ds] > 0$, i.e. the margin account should hold enough collateral to pay for the streaming premia in the considered future period of time. Despite the fact that ES_λ satisfies the subadditive property $\mathrm{ES}_\lambda[X+Y] \leq \mathrm{ES}_\lambda[X] + \mathrm{ES}_\lambda[Y]$, Figure 4 provides an intuition on how the collateral is decomposed and the offset of the risks between the underlying and the streaming premia.

The three considered options have limited downside, hence $\mathrm{ES}_\lambda[V(S_1) - V(S_0)]$ will be small. In the particular case of the strangle option, the GPH will have to pay the premium fee whenever the price crosses the two ranges $[9, 10)$ and $[11, 12)$, therefore the collateral provided by the GPH in this case will have to be bigger than in the ITM / OTM call examples, where the GPH has to pay a premium fee when the price only crosses one range.

We observe in Figure 4 that there appears to be negligible offsetting of the risks between the underlying and the streaming premia – in the examples considered, one can compute the margin requirements separately for the streaming premium and the underlying payoff and sum them. This is due to a degree of correlation between the payoff and the streaming premium, which removes any

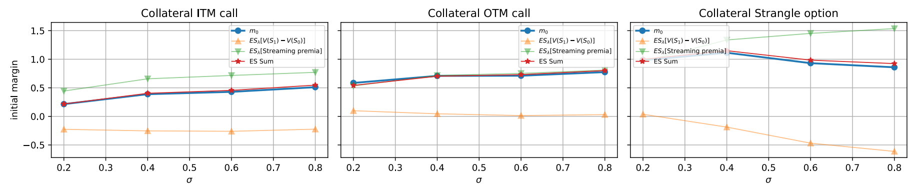


Figure 4: Collateral m_0 in blue, $\mathrm{ES}_\lambda[\eta(S_1) - \eta(S_0)]$ in orange $\mathrm{ES}_\lambda[-\int_0^1 \hat{g}_s^* ds]$ in green for the first three positions, and for different values of σ.

beneficial hedging effect. We see that the risk associated with the streaming premium is a significant factor, as we see in Figure 5, where we plot the distribution of the streaming premia for the Strangle option payoff given in Figure 4, in the case where $\sigma = 0.4$. This suggests that the main risk management concern, for convex position holders, is often being able to guarantee payment of the streaming premium.

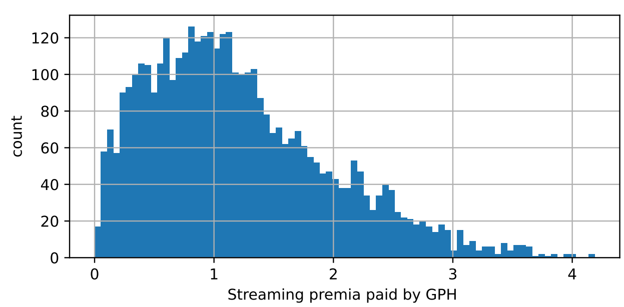


Figure 5: Distribution of the streaming premia for the Strangle option payoff

8.1.2 Initial margin account of general position and short-put holders

We consider in this subsection the initial margin account of the positions that involve moving liquidity from the Panoptic pool to Uniswap. These correspond to Figures 3d, 3e and 3f. In our simulations, we also consider the presence of a general position holder that pays some streaming premia to utilise the liquidity of the short-put holder to open their position.

Figure 6 shows the collateral needed by the GPH to open their position. The expected shortfall is calculated with $\lambda = 0.9$ and fixed Uniswap fee 0.3%. As before, we also plot $\mathrm{ES}_\lambda[\eta(S_1) - \eta(S_0)]$ and $\mathrm{ES}_\lambda[-\int_0^1 \hat{g}_s^* ds]$.

As before, we observe in Figure 6 that there appears to be negligible offsetting of the risks between the underlying and the streaming premia – in the examples considered, one can compute the margin requirements separately for the streaming premium and the underlying payoff and sum them.

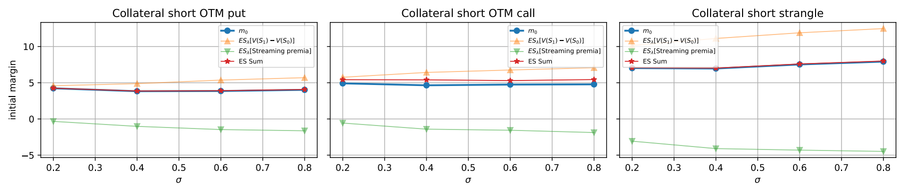


Figure 6: Collateral m_0 in blue, $\mathrm{ES}_\lambda[\eta(S_1) - \eta(S_0)]$ in orange $\mathrm{ES}_\lambda[-\int_0^t \hat{g}_s^* ds]$ in green for the three positions, and for different values of σ.

Interestingly, the simulations show little correlation between the underlying volatility and the expected shortfall of the position. This indicates that the eventual margin calculations might not require an approximation of the volatility of the underlying asset in some cases (even though this quantity will be needed for calculation of the streaming premium).

8.1.3 Panoption fee income (SB3) in OTM call - High and low fee scenarios

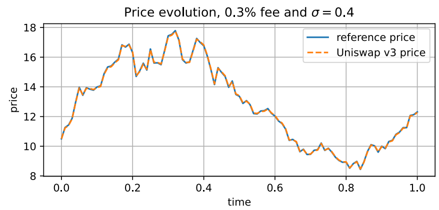
We run simulations with and without the the presence of a GPH, i.e. with and without the Panoption fee income (SB3). The goal is to evaluate the incentives for a short-put holder to open a position (deposit liquidty to CFM through Panoptic) compared to directly depositing liquidity to the CFM without the Panoptic intermediate step.

In this section we consider the fees due to a short-put holder, and how the presence of the GPH (a long-put holder) changes their income. We consider the position with payoff as in Figure 3b.

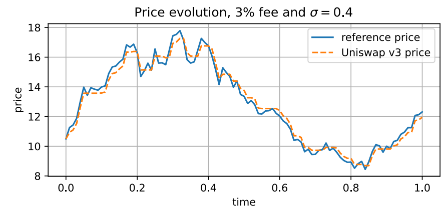
Figure 7 shows the the reference price evolution and the CFM price evolution for one simulation for low CFM fee (0.3%) and high CFM fee (3%). The smaller the fee (the higher the value of γ) the closer the CFM price process depicted in orange follows the reference market price S_t.

For these two scenarios the CFM and the GPH non-arbitrage fee rates are shown in Figure 8 with and without GPH (blue and orange respectively). The area between both curves provides the additional fee income that the short-put holder receives under the presence of the GPH. Figure 8b indicates that under a high CFM fee regime most of the short-put holder fee income comes from the arbitrageur's trades but nevertheless it is still preferable for the short-put holder to provide liquidity to Uniswap v3 through the Panoptic protocol.

Figure 9 aggregates the results of the 1000 simulations for each considered γ, σ. Figure 9a confirms that in our current setting, the short-put holder has incentives to open a position and these incentives increase with the volatility of the risky asset, and are not impacted by the Uniswap v3 fee.

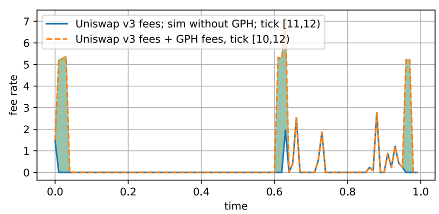


(a) Evolution of CFM price and reference price in low fee simulation 0.3%

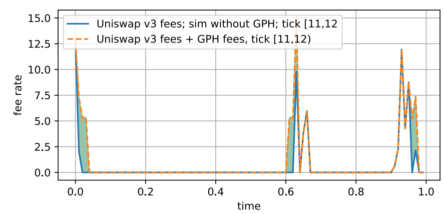


(b) Evolution of CFM price and reference price in high fee simulation 3%

Figure 7: Evolution of CFM price and reference price in low (left) and high (right) fee settings



(a) Evolution of fee rate with and without a GPH with low CFM fee 0.3%.



(b) Evolution of fee rate with and without a GPH with low CFM fee 4%.

Figure 8: Evolution of fee rate with and without a GPH under different CFM fee scenarios. The area between the curves indicates the accumulated fee received by the short-put holder when a GPH opens a position. Note that the Uniswap v3 fee and the GPH streaming premia are non-zero only when the price S_t crosses the range $[11, 12)$, which is where the short-put holder deposited their liquidity and where the GPH opened their position.

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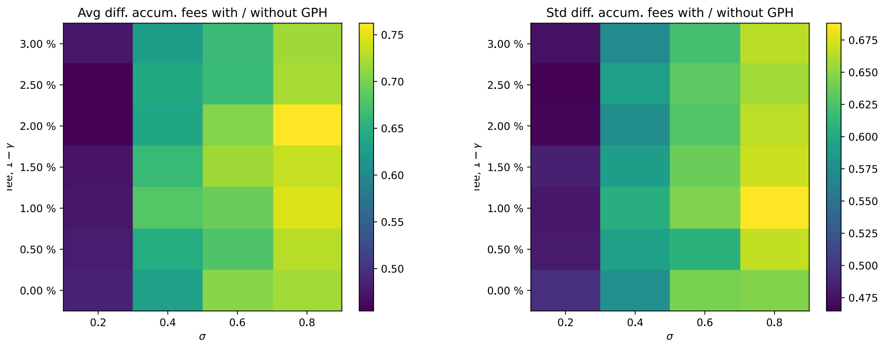

(a) Mean difference of accumulated fee income between simulations with GPH and simulations without GPH

(b) Standard deviation of difference of accumulated fee income between simulations with GPH and simulations without GPH

Figure 9: Mean and standard deviation of difference of accumulated fee income between simulations with GPH and simulations without GPH for different values of γ, σ.

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A Payoffs of position holders

In this section we provide more details of the positions taken by the general position holders to achieve the studied payoffs.

- Convex payoffs, achieved by the general position holder (GPH) position:

  a) ITM call: the GPH withdraws some liquidity from a price range below the current risky asset price. In addition, the GPH goes long in the risky asset.

b) OTM call: the GPH withdraws some liquidity from a price range above the current risky asset price.

c) Strangle: The GPH withdraws liquidity from two price ranges above and below the current risky asset price.

- Concave payoffs, achieved by the short-pull holder position.

  a) Short OTM put: the short-put holder moves liquidity from the Panoptic pool to the Uniswap pool in a price range below the current risky asset price.

  b) Short OTM call: the short-put holder moves liquidity from the Panoptic pool to the Uniswap pool in a price range above the current risky asset price.

  c) Short Strangle: the short-put holder moves liquidiry from the Panoptic pool to the Uniswap pool in two price ranges above and below the current risky asset price.

  Note that in the above positions, a GPH makes the opposite moves (with some utilisation percentage), so that the short-put holder receives some streaming premia from the GPH for using their liquidity.

B Agent-based model - Panoptic and Uniswap v2

We turn into numerical simulations to study the Streaming Premia SB3, GB3 in different scenarios of low / high price volatility and low / high CFM fee $(1-\gamma)$. The considered CFM models Uniswap v2.

We consider three agents, a) a short-put holder who moves assets from the Panoptic pool into Uniswap, b) a general position holder who moves a portion of the assets back in the Panoptic pool, and pays a streaming premia to compensate for the Uniswap fee income the short-put-holder would receive had they not entered the general position. And c) an arbitrageur who trades between the CFM and a reference market to make risk-free profits.

B.0.1 Short-put holder and CFM

We consider a CFM as in Example 3.3 with $\theta = 1/2$, x^1 the numeraire. Initial reserves (x_0^1, x_0^2) are entirely provided by a short-put holder through the Panoptic pool. The CFM trading function satisfies $\Psi(x_t^1, x_t^2) = x_t^1 \cdot x_t^2 = (\kappa_t)^2$, where κ_t stays constant as long as no liquidity is deposited in / withdrawn from the pool.

B.0.2 General position holder

At initial time a general position holder (denoted by GPH) withdraws a portion $\beta \in (0,1]$ of the CFM liquidity. There reserves left in the CFM satisfy the equation $\Psi(x_{t_k}^1, x_{t_k}^2) = \beta\kappa$

When the position is closed, the GPH gets a payoff

$$\eta_{S_{t_k}}(S_T) = \frac{1}{2}(1-\beta)\kappa S_{t_k}^{1/2} - \frac{1}{2}(1-\beta)\kappa S_T^{1/2},$$

which is a convex function in S_T. In other words, the GPH has actually purchased a perpetual American option with payoff $\eta_{S_{t_k}}(\cdot)$. The theoretical non-arbitrage streaming fee using a delta hedge is then

$$\hat{g}_t = -\frac{1}{2}\sum_{i,j=1}^{2} \partial_i\partial_j \eta_{S_{t_k}}(S_t)\frac{d\langle S^i, S^j\rangle_t}{dt} - \Big(\sum_{i=1}^{2}\partial_i \eta_{S_{t_k}}(S_t) S_t^i\Big)\frac{d\log B_t}{dt}.$$

Note that under zero risk-free rate, $\hat{g}_t$ is negative because of the convexity of $\eta_{S_{t_k}}(\cdot)$. Hence $\hat{g}_t$ can be seen as the rate fee paid by the GPH to the short-put holder, the issuer of the perpetual American option.

B.0.3 Arbitrageur

Price discovery occurs in a second reference market, where S_t denotes the price of asset x^2 in terms of the numeraire x^1 at time t. We model the price S_t by a Geometric Brownian Motion

$$dS_t = \sigma S_t dW_t, \quad S_0 > 0.$$

Whenever the price of asset x^2 in terms of the numeraire x^1 in the CFM is different than S_t, there might be an arbitrage opportunity. An arbitrageur will automatically make the necessary trades to maximise their profit. The arbitrageur who trades $(\Delta x^1, \Delta x^2)$ when reserves are (x^1, x^2) will incur cost $f(\gamma, x^1, x^2, \Delta x^1, \Delta x^2)$ and solves the optimisation problem

$$\Delta x^{2,*} := \max(\Delta x^2_{\text{CFM, CEX}}, \Delta x^2_{\text{CEX, CFM}}),$$

where

$$\begin{aligned}\Delta x^2_{\text{CFM, CEX}} :=& \arg\max_{\Delta x^2} \Delta x^1 \cdot S^1 - \Delta x^2 - f(\gamma,, x^1, x^2, \Delta x^1, \Delta x^2)\\ &\text{such that } \Psi(x_1 - \Delta x^1, x_2 + \Delta x^2) = \Psi(x_1, x_2), \quad \Delta x^2 \geq 0,\\ &\Delta x^1 \cdot S^1 - \Delta x^2 - f(\gamma, x^1, x^2, \Delta x^1, \Delta x^2) \geq 0\end{aligned}$$

and

$$\begin{aligned}\Delta x^2_{\text{CEX, CFM}} :=& \arg\max_{\Delta x^2} \Delta x^2 - \Delta x^1 S^1 - f(\gamma, x^1, x^2, \Delta x^1, \Delta x^2)\\ &\text{such that } \Psi(x_1 + \Delta x^1, x_2 - \Delta x^2) = \Psi(x_1, x_2), \quad \Delta x^2 \geq 0,\\ &\Delta x^2 - \Delta x^1 S^1 - f(\gamma, x^1, x^2, \Delta x^1, \Delta x^2) \geq 0\end{aligned}$$

with the subindexes of $\Delta x^2_{\cdot,\cdot}$ denoting the order of the markets where the arbitrageur goes short and long in asset x^2. If $\Delta x^{2,*} = 0$, then there is no possible trade for which an arbitrageur makes a profit.

B.0.4 Fee specifications of the simulation

Following the fee specifications in Section 6.1 and focusing in SB3, GB3, the agents will receive / pay the following:

a) **Short-put holder**:

- For $t \in [0, T)$ receipt of CFM fee income given by $(1 - \gamma)$ paid by the arbitrageur's trades.
- For $t \in [t_k, T]$ receipt of the Panoption streaming premia paid by the GPH (denoted by SB3 in Section 6.1).

b) **General position holder**

- For $t \in [t_k, T]$ payment of the Panoption streaming premia to the short-put holder (denoted by GB3 in Section 6.1).

c) **Arbitrageur**

- For $t \in [0, T]$, payment of the CFM fee to the short-put holder.

B.1 Simulation

We run the simulation of discrete times $\Pi := \{0, t_1, t_2, ..., T\}$ and initialise the simulation with reserves (x_0^1, x_0^2) and fee $(1 - \gamma)$; initial price S_0 and volatility σ. We run the following simulation.

For every $t \in \Pi$:

1. If $t = 0$, the short-put holder moves the liquidity from the Panoptic pool to the CFM
2. The arbitrageur makes the necessary trades resulting from optimisations in the CFM and in the reference market to make a risk-free profit, and pays the corresponding CFM fee to the short-put holder.
3. If $t = 0.5$, a GPH withdraws 10% of the CFM liquidity and puts it back in the Panoptic pool. If $t > 0.5$, the GPH pays the premium fee to the short-put holder. If $t = 1$ the GPH closes their position and moves the necessary liquidity from the Panoptic pool to the CFM pool.

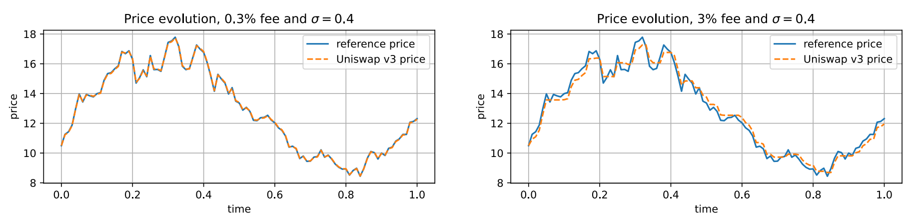
We run simulations with and without the blue step to evaluate the fee income of the short-put holder with and without the presence of a GPH, i.e. with and without the Panoption fee income (SB3). The goal is to evaluate the incentives for a short-put holder to open a position (deposit liquidty to CFM through Panoptic) compared to directly depositing liquidity to the CFM without the Panoptic intermediate step.

We take $\sigma \in \{0.2, 0.4, 0.6, 0.8\}$ and fix $S_0 = 10.5, x_0^1 = 100, x_0^2 = 10$ and run the above simulation 100 times for different values of $\gamma \in [0.96, 1]$.

B.1.1 High and low fee scenarios

Figure 10 shows the the reference price evolution and the CFM price evolution for one simulation for low CFM fee (0.3%) and high CFM fee (4%). The smaller the fee (the higher the value of γ) the closer the CFM price process depicted in orange follows the reference market price S_t. For the first low CFM fees scenario, Figure 11 provides the reserves evolution (x_t^1, x_t^2) with and without GPH (blue and orange respectively). When the GPH is included in the simulation they withdraw 20% of the CFM liquidity at time $t = 0.5$ and close their positions at time $t = 1$.

For these two scenarios the CFM and the GPH non-arbitrage fee rates are shown in Figure 12 with and without GPH (blue and orange respectively). The area between both curves provides the additional fee income that the short-put holder receives under the presence of the GPH. Figure 12b indicates that under a high CFM fee regime and for some of the arbitrageur's trades the short-put gets penalised for not having all their liquidity in the CFM and the GPH premium fee cannot compensate for it.



(a) Evolution of CFM price and reference price in low fee simulation 0.3%

(b) Evolution of CFM price and reference price in high fee simulation 4%

Figure 10: Evolution of CFM price and reference price in low (left) and high (right) fee settings

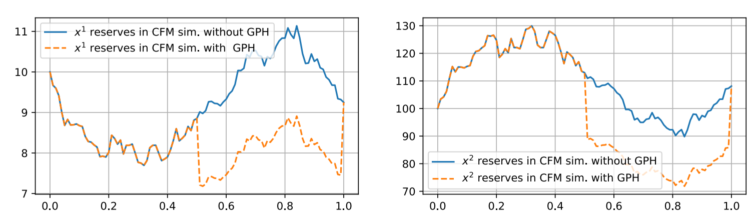
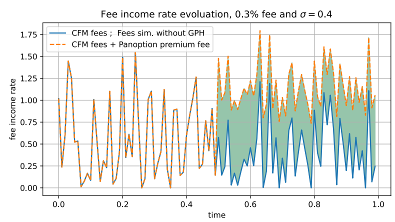
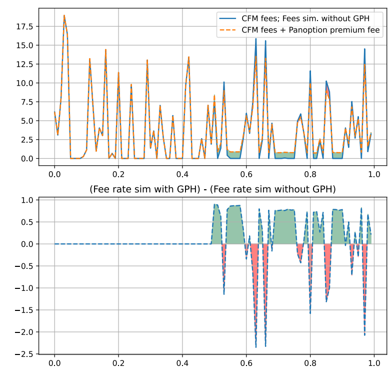


Figure 11: Evolution of CFM reserves in low fee simulation 0.3% with and without GPH



(a) Evolution of fee rate with and without a GPH with low CFM fee 0.3%.

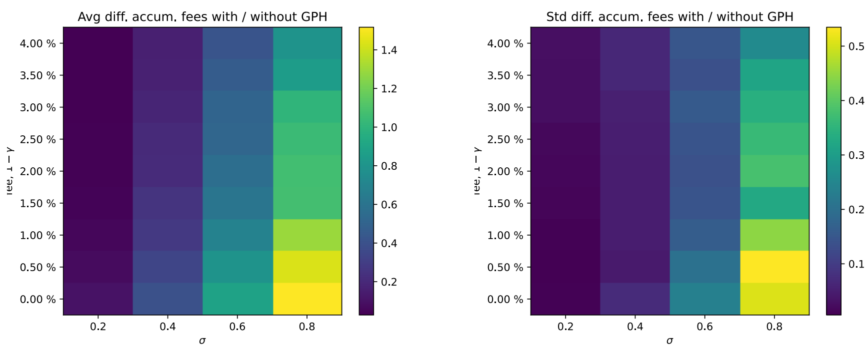


(b) Evolution of fee rate with and without a GPH with low CFM fee 4%.

Figure 12: Evolution of fee rate with and without a GPH under different CFM fee scenarios. The area between the curves indicates the accumulated fee received by the short-put holder when a GPH opens a position.

B.1.2 Aggregated results

Figure 13 aggregates the results of the 100 simulations for each considered γ, σ. Figure 13a confirms the intuition from the the two examples in the Figure 12. In our current setting, the short-put holder has incentives to open a position in a low fee CFM regime, which is in fact a realistic assumption, and these incentives increase with the volatility of the risky asset.



(a) Mean difference of accumulated fee income between simulations with GPH and simulations without GPH

(b) Standard deviation of difference of accumulated fee income between simulations with GPH and simulations without GPH

Figure 13: Mean and standard deviation of difference of accumulated fee income between simulations with GPH and simulations without GPH for different values of γ, σ.